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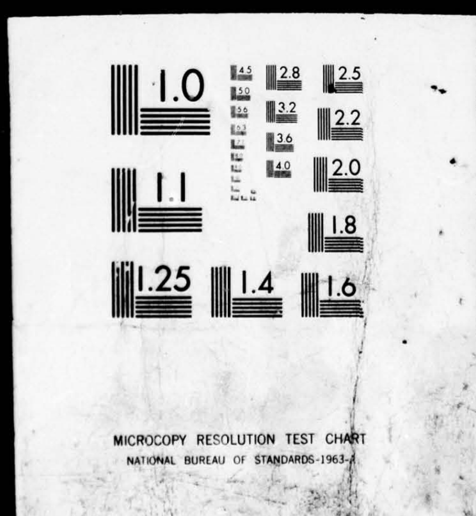

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A FAST SOLUTION OF THE LINDLEY EQUATIONS FOR THE M-GROUP REGRESSION PROBLEM

BY

IVO W. MOLENAAR

RIJKSUNIVERSITEIT GRONINGEN

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This paper presents a solution to the Lindley equations (Lindley and Smith, 1972) for an m-group regression problem. The present model is more efficient than earlier versions from the point of view of both computer CPU time and memory. The present solution lends itself to implementation on smaller computer systems. An explanation of the solution is presented, as well as directions for its implementation.

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A FAST SOLUTION OF THE LINDLEY EQUATIONS

FOR THE M-GROUP REGRESSION PROBLEM

Ivo W. Molenaar
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ABSTRACT

This paper presents a solution to the Lindley equations (Lindley and Smith, 1972) for an m-group regression problem. The present model is more efficient than earlier versions from the point of view of both computer CPU time and memory. The present solution lends itself to implementation on smaller computer systems. An explanation of the solution is presented, as well as directions for its implementation.

A FAST SOLUTION OF THE LINDLEY EQUATIONS FOR THE M-GROUP REGRESSION PROBLEM*

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0. Summary

This report is based on a discussion by Jones and Novick (1972) of the technical problems involved in obtaining Bayesian modal estimates for the regression parameters in m similar groups. A general discussion of Bayesian linear model analysis is found in Lindley and Smith (1972). The methodology of m-group regression is explained in Jackson, Novick, and Thayer (1971). Cross-validation is reported by Novick, Jackson, Thayer and Cole (1971, 1972), and Jansen (1977); Shigemasu (1976) presents a solution and cross-validation for the case of equal slopes and unequal intercepts.

The available computer programs, BPREP-BASIC and BAYREG, both written in FORTRAN, require an amount of core and CPU that does not encourage regular use. These programs have been analyzed with the aim of suggesting improvements that might bring the performance of algorithms to a point where inclusion in the CADA Monitor (Novick, Isaacs, and DeKeyrel, 1977), after conversion into BASIC, could become feasible. Slight modifications to the preparatory program BPREP,

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and major changes to BAYREG were made, resulting in a speeded-up algorithm. Compute time (both on the IBM 370 VS in Iowa City and on the CDC Cyber 74/18 in Groningen) was reduced by a factor of 2 to 4 in four benchmark examples, which are believed to be fairly representative of the type of multiple regression problems where Bayesian m-group regression might be used. However, it is advisable to use the speeded algorithm for some more data sets and build up more experience about its optional parameters, before the program is included in the CADA Monitor.

Section 1 of this report can be read as an update to the input description in Jones and Novick (1972), giving the user sufficient information for the use of the modified programs. Section 2 describes the deficiencies of the old programs BPREP and BAYREG, and briefly outlines the improvements that have been made in the new program MBREG. Section 3 contains the detailed proposal for a revision of BPREP into a new preparatory program MPREP adapted to the new main program MBREG; this revision will not require substantial programming efforts. Section 4 is devoted to the explanation of the "leaps" which constitute the major improvement of MBREG over BAYREG. The performance of the improved program for five data sets is described in section 5. The final section 6 mentions a few points deserving further investigation; some of these could be settled before the CADA Monitor version of the program is written.

1. Revised Input Description

Running a Bayesian m-Group Analysis

The data decks for MPREP and for MBREG are quite similar, since the data deck for MPREP is a subset of the deck for MBREG. Accordingly, the data deck for MBREG is described below:

1. Identification Card (10A8)

Columns	Name	Description
Col. 1-80		Identification for data

2. Parameter Card (3I4, 3F5.0, 7I2, 3F5.1)

M and NV must be read in, other parameters get default values if blank.

Columns	Name	Description
Col. 1-4	M	Number of schools (maximum 25)
5-8	NV	Number of predictors (maximum 4)
9-12	NCY	Number of cycles (maximum 100, default = 30)
13-17	CKAPPA	Small Constant (default = .001). The constant κ in the equations is taken as CKAPPA/THETA, where THETA is the harmonic mean of the $\hat{\phi}_i$ (after replacing $\hat{\phi}_i$ < PHIMIN by this lower bound)
18-22	DCON	λ = DCON τ (default = 3). The diagonal of Σ in the equations contains DCON * (a priori estimates of τ_α and $\tau_{\beta h}$ given on card 9)
23-27	PHIMIN	Minimum $\hat{\phi}_i$ allowed (can be arbitrarily small, default = .001)
28-29	IWR	0 - No details on iteration (default) 1 - Criterion and all values printed after each cycle
30-31	INIT	0 - Starting values in original scaling of card 3 (default) 1 - Starting values in ideal scaling 2 - Starting values scaled at origin
32-33	IPUN	0 - No punched output (default) 1 - Bayesian modal estimates are punched (8X, 6E12.6)
34-35	NDH	Iteration stops when criterion constant in NDH digits (default = 5)
36-37	NDB	Iteration stops when all α_i, β_{hi}, and ϕ_i constant in NDB digits (default = 4); if relative change is less than 10^{-NDB} no leap is taken for that estimate
38-39	NCI	Number of cycles before first leap (minimum 4, default = 5)
40-41	NCF	Number of cycles between leaps (minimum 4, default = 4)

2. Parameter Card (con't.)

Col. 42-46	SCH	Leap = SCH * last difference if difference has just changed sign or old difference almost equal to 0 (default = 20.0)
47-51	DCN	Leap = DCN * last difference if this difference is not substantially closer to zero than previous difference (default = 10.0)
52-56	VGT	At first decrease of criterion not after leap or start, next cycle starts at VGT * old + (1 - VGT) * new values (default = .5)

3. Predictor Card (4A8)

Col. 1-8	Name of 1st predictor
9-16	" " 2nd "
.	.
.	.
.	.

4. Scaling Card for Original Scaling Points (5F8.0)

Col. 1-8	Value to which criterion has been scaled
9-16	Value to which predictor 1 has been scaled
17-24	" " " " 2 " " "
.	.
.	.
.	.

5. Scaling Card for Ideal Points (5E13.6)

Col. 1-13	Value to which criterion has been scaled
14-26	Ideal scaling point for predictor 1
27-39	" " " " " 2
.	.
.	.
.	.

6. Format Card for SCP Matrix (A8)
   The cross products must be read in floating point form.

7. SCP Matrix Cards

For each group, there must be an upper triangular cross-product matrix punched according to the format specified by card 6. The cross-product matrices have the following form for the case of two predictors:

$$\begin{array}{l} \text{Row 1} \\ \text{Row 2} \\ \text{Row 3} \\ \text{Row 4} \end{array}
\begin{bmatrix} n_i & \Sigma x_{i1j} & \Sigma x_{i2j} & \Sigma y_{ij} \\ \Sigma x_{i1j}^2 & \Sigma x_{i1j}x_{i2j} & \Sigma x_{i1j}y_{ij} & \\ \Sigma x_{i2j}^2 & \Sigma x_{i2j}y_{ij} & & \\ \Sigma y_{ij}^2 & & & \end{bmatrix}$$

Comparison of this matrix with formula (5) reveals that rows 2, 3, and 4 have been translated to the left. These cross products are scaled to the values given by card 4.

8. Coefficient cards (6E12.6)

For each group, there must be an initial coefficient vector which consists of $(\alpha_i, \beta_{1i}, \ldots, \beta_{\ell i}, \phi_i)$. The α_i is for the data coded to original scaling, ideal scaling or scaling at origin for INIT = 0, 1, 2 respectively. In most cases the coefficient cards will contain the least square coefficients per group, produced by MPREP or obtained separately.

9. Population Variance Estimates Card (6E12.6)

See Jones and Novick (1972, pp. 17-18). The estimates for population variances are the components of $(\hat{\tau}_\alpha, \hat{\tau}_{\beta 1}, \ldots, \hat{\tau}_{\beta \ell})$. Typically $\hat{\tau}_\alpha$ and $\hat{\tau}_{\beta h}$ are values computed by the MPREP program; if some of the original estimates were negative, then these estimates have been replaced by a small positive number. The value $\hat{\tau}_\alpha$ (at ideal point) for asymptotic values has then been recalculated.

The data deck for MPREP differs from that for MBREG in the following ways. The scaling card for ideal points is not included nor are the coefficient cards or the population statistics (all these cards result from the MPREP analysis). In summary, the respective items in the data decks are listed below.

	MPREP	MBREG
1. Identification Card	Yes	Yes
2. Parameter Card	Yes	Yes
3. Predictor Card	Yes	Yes
4. Scaling Card for Original Scaling Points	Yes	Yes
5. Scaling Card for Ideal Points	No	Yes
6. Format Card for SCP Matrix	Yes	Yes
7. SCP Matrix Cards	Yes	Yes
8. Coefficient Cards	No	Yes
9. Population Variance Estimates Card	No	Yes

Items 8 and 9 are provided as part of the punched output from MPREP. Item 5 is the first card of this output. (In the earlier version it was punched by the investigator from the printed output from MPREP and inserted in the data deck.

Several bits of information required in items 1-9 can be placed in the context of Bayesian methodology. The constant CKAPPA bounds the posterior density away from the point at which the residual variances are

equal. The value DCON is a factor which inflates the estimates of τ_β; it formally equates the mode of the prior to the value specified a priori (cf. Jackson, Novick, and Thayer, 1971, p. 131). PHIMIN places a lower bound on the residual variance estimate for any given group.

Often it is desirable to set the value of IWR to one, since the investigator will then be able to observe the computer's search for the mode of the posterior distribution. (The reason that the log of the height function is printed rather than the actual height is that storage of the actual height might cause problems within the computer when the height is very small.) As the iteration process converges, the log of the height function will increase to the modal value and then exhibit very small fluctuations about this value.

The scaling values for the original scaling points (item 4) may be all zero (indicating lack of rescaling) if desired; however, if the investigator uses variables with large means, he may wish to scale the variables to their approximate means before producing the SCP matrix. To do this, he simply subtracts a constant from each individual's score. Such a procedure will tend to reduce the magnitude of the numbers contained in the SCP matrix and may decrease the chances for rounding errors in a problem involving large sample sizes. A different scaling point, of course, may be selected for each variable.

The original scaling points should be distinguished from the ideal scaling points. The ideal scaling points are not a mere matter of con-

venience; they are intrinsic to the Bayesian method being discussed. Also, the ideal points apply only to the predictor variables, whereas the original scaling points are relevant to the predictors and to the criterion.

We punch the symmetric SCP matrix in a special upper-triangular form. The first row contains the full number of $(\ell + 2)$ elements. If necessary, the punching continues on subsequent cards according to the format card. The second row must begin on a new card but omits the first element in order to begin with the diagonal elements in the second row; thus, the punched form of the second row has only $(\ell + 1)$ elements. Continuing in this fashion, we find that the last rwo, when punched, contains only the diagonal (last) element.

We next present some recommendations for the input parameters governing the leap process which is discussed in more detail in section 4. When a user has no special information on his m-group regression data set, it is recommended that he start running the program with all default options in vigor. The log posterior density is printed after every cycle, and gives an impression of the convergence of the estimation process. When desired, the option IWR = 1 leads to the printing of criterion and parameter values at every cycle (one line per group per cycle), which permits a detailed investigation of the behavior of the individual parameters during iterations and leaps.

If these results are not satisfactory, it is recommended to look in some detail at the first leap. If this leap is based for a majority of

parameter values on differences which exhibit a sign change or an increase in absolute value (see examples in section 4), it may be wiser to postpone the first leap for a few cycles by choosing a larger value for NCI. When it appears that the leaps are too small (i.e., the iterations following it move the estimates in the same direction); then it may help to increase SCH and/or DCN, provided that at least one leap followed after a sign change or increase of differences. Similarly a low value for DCN and/or SCH could be inserted if the leaps are too large (are followed by movements in the opposite direction). It should be stressed that these manipulations have a direct effect on the estimates leaping after sign change or increase of differences, but also an indirect effect, via the mean across groups, on all other estimates. The very large value DCN = 30.0 was successful for three of the data sets mentioned in section 5 for just this reason: it produced a too large leap for the relatively few parameters where the difference had not decreased, but in the next iteration step this pulled all the remaining estimates which had leapt not far enough by the geometric ratio extrapolation, in the right direction more than an ordinary iteration would have done. As this phenomenon might be unusual, we have kept the safer value DCN = 10 as default value.

Part of the success of large DCN values is explained by the rule in the program that an estimated geometric ratio ρ (see section 4) for which $\rho/(1 - \rho)$ exceeds DCN is replaced by DCN. This rule prevents gigantic overshoots for cases where a ratio $\rho = .98$, say, is produced at a point

where the ratio of successive differences is not sufficiently stable: the function $\rho/(1 - \rho)$ becomes too steep for ρ near 1.

A comment should be made on the double role of NDB. The program stops when all estimates undergo a relative change of less than 10^{-NDB}. At the same time, any estimate for which the last difference was that small is excluded from the leap process. This helps to prevent instability; leaps that are based on insignificant changes in the estimates could do more evil than good. In our experience a stopping rule based on relative changes, unavoidable because the order of magnitude of the estimates will often be unknown, has the undesirable side effect that one incidental value very close to 0, typically an intercept, will cause the process to continue when a user might be satisfied with the precision of the estimates. This holds especially when high precision is desired. Moreover, optimal speeding of the leap process is sometimes impeded by a choice NDB = 2 or NDB = 3 which might work well in the stopping rule. Perhaps the double role should be revised when more experience on other data sets has been collected.

The undisturbed iteration process, without leaps or reduced steps, is obtained when the user specifies values NCT $\geq$ NCY, NCI $\geq$ NCY and VGT = 0.0. A detailed study of this process (IWR = 1) may be desirable when the speeded solution process does not produce satisfactory results. Such cases have not yet been encountered, and the "bad leap" provision already offers protection against occasional malfunctioning, but it may

still be worthwhile to know that the original slow convergence process, without leaps and without other stopping rules than NCY = total number of cycles, is still available as a subset in the present program.

By putting IPUN = 1, the user obtains the best solution in punched cards. This may be useful when a restart from this solution with other leap-governing parameters is desired; it also saves computer time in cases where the process might turn out to be "on its way" without having achieved yet the amount of convergence required by the user.

2. Conclusions, Deficiencies and Changes in the Programs

Changes were made in BAYREG and proposed for BPREP with the double aim of speeding the execution and reducing the core requirements. Study of both programs in their July 1977 form led to the following conclusions:

1. BAYREG either uses two sets of initial estimates, least squares (LS) and asymptotic (ASY, essentially Model II analysis of variance estimates; see Jones and Novick, 1972, p. 18), or uses read-in initial estimates.
2. The final Bayesian estimates maximizing the criterion (which is the logarithm of the posterior density) are typically closer to ASY than to LS, and the criterion value for ASY is higher than for LS. Nevertheless, iteration from LS starting values produced in all benchmark examples a sequence of criterion values which overtook the sequence starting at ASY values, sometimes at the third sometimes at the fifth

cycle. Criterion values from LS starting remained consistently higher during the further iteration process. Novick, Jackson, Thayer, and Cole (1971, p. 27) have also started iteration from LS values.

3. In the iterative solving of the Lindley equations, BAYREG uses as prior values three times the estimated population variances τ_α (of the intercept) and $\tau_{\beta h}$ (of the h^{th} predictor, $h = 1, 2, \ldots, \ell$). It takes those as calculated and punched in BPREP, but changes negative $\tau_{\beta h}$ values into 10^{-7} and then adapts τ_α to such changes. This leads to totally regressed β_{hi} values for all h for which $\tau_{\beta h}$ was negative and to changes in τ_α and α_i values. This modified ASY solution is used as a starting point for the iteration.
4. BAYREG contains dimension statements for some arrays not used at all, and frequently uses two or more arrays where one array could serve for more than one purpose. Moreover, the chaotic numbering of the statements and the use of parallel names for essentially the same variable afflicts the transparency of the program. It is clear that the program could use a "cleaning" after frequent adhoc changes.
5. Either immediately or after a few cycles, the iteration process typically consists for many cycles of taking too small steps in the right direction.
6. After finally reaching a kind of plateau value for the criterion, the iteration process shows an oscillatory behavior, in which a step leading to a small decrease of the criterion is followed by some steps bringing it back to roughly the best value obtained thus far. Near its maximum, the criterion seems to be a very flat function of its many variables.

details on the necessary revision of MPREP are given in section 3. This preparatory program should be adapted to produce all the punched output that a user might need for the input data deck to MBREG. In the stage of insertion into the CADA Monitor, the preparatory program and the MBREG program can be turned into two separate modules with a division point at which only a limited data transfer is required and with a more logical division of tasks between the two modules. Some remarks on the use and specification of the prior variances will be made in section 6 of this report.

C. The program BAYREG has been completely rewritten, in a more accessible notation involving less arrays and variables. From 14401 array elements in BAYREG there are left 2109 elements in the rewritten program MBREG; the number of FORTRAN lines was reduced from 429 to 336, including many more comment lines than before. This will mean faster compilation, less core requirements, and faster execution. Part of this economy is achieved by reducing the maximum number of predictors from 10 to 4 and the maximum number of groups from 40 to 25. This harmonizes with the pleas of Jones and Novick (1972, pp. 10-11) and Jackson, Novick, and Thayer (1971, p. 139). The revised program is designed to make further reduction or extension of these limits, when desired, as easy as possible.

D. The slow climbing mentioned in conclusion (5) is speeded up by taking a "leap" to values extrapolated from the values at the past three cycles. Such leaps are taken every NCF cycles (typically 5 or 4), with an initial leap after NCI cycles. The leap process is governed by several parameters described in sections 1, 4, and 5. Default values for these parameters are provided by the program, but the user may provide other values when the default values do not lead to a satisfactory convergence of the iteration process. The graphs given in section 5 clearly show for some benchmark examples that the introduction of such leaps reduces the required number of iterations by a factor of at least 3.

E. Conclusion (6) has led to the introduction of a special provision for any iteration which would lead to a decrease of the criterion to be maximized, which is log posterior density. The parameter values to be maximized are stored before the decrease and the step is reduced to 1 - VGT times its size (VGT = .5 is usual). If iterations resumed from this point in the parameter space do not improve upon the maximum criterion value thus far within the next two cycles, the process stops and the maximizing values are printed. Otherwise, the iteration process continues but no leaps are undertaken until NCF iterations have been made from the values reached by the reduced step.

F. The process stops after:

- a failure of improvement occurs as described under (E);
- all parameter values undergo a relative change of less than 10^{-NDB} (remain constant in NDB leading digits);
- the criterion undergoes a relative increase of less than 10^{-NDH} (remains constant or decreases in NDH leading digits) during several cycles, preceded by a leap;
- a maximum of NCY cycles has been carried out (default value NCY = 30, maximum NCY is 100; inspection and revision of the iteration process is believed to be more fruitful than prolonged ineffective iterations).

In all cases, the best modal estimates obtained thus far are printed (and also punched when desired).

G. The (alphabetical) ordering "alpha, beta, phi" is systematically used in all inputs and outputs.

H. It will be recommended in section 6 to make the CADA Monitor version of the program more fool-proof with respect to conclusion (7).

3. Changes in the Preparatory Program BPREP

As documented in Jones and Novick (1972), the preparatory program in its present form serves three purposes:

(1) It calculates the "ideal scaling points," i.e., the values at which the predictors must be centered in order to make them a priori uncorrelated with the intercept. These ideal scaling points and the estimated predictor variances and covariances between intercept and predictors are printed.

(ii) It calculates the least squares (LS) estimates of regression coefficients and intercepts, together with their estimated squared errors, and the estimated residual variances. These are all punched. The printed output omits the estimated squared errors, and adds the intercept values for predictors scaled at the origin, the scaling point, and the ideal point.

(iii) It calculates the asymptotic (ASY) estimates, essentially Model II analysis of variance estimates, of the regression coefficients, the intercepts, and the residual variances. The variances of the regression coefficients, calculated across groups, involve both LS and ASY estimates as detailed by Jones and Novick (1972, pp. 17-18). The estimates $\hat{\tau}_{\beta h}$ obtained in this way can be negative; they are printed and punched together with $\hat{\tau}_{\alpha}$.

In the July 1977 form of BAYREG, from statement 99 onward, these results are used as follows:

a. Possible negative estimates $\tau_{\beta h}$ are replaced by 10^{-7}. (The lower bound of 10^{-4} given by Jones and Novick, 1972, p. 18, can be too high to produce the almost total regression of the coefficients β_h; in our opinion the lower bound should be made dependent on the order of magnitude of the coefficients β_h themselves, because a general purpose program should be robust against rescaling of predictors or criterion, see also section 6.)

b. Possible residual variance estimates $\hat{\phi}_i$ less than PHIMIN are replaced by PHIMIN (a lower bound which can be arbitrarily small, according to Jones and Novick, 1972, pp. 22 and 25). We shall point out in section 6 that such a bound should be robust against rescaling.

c. The ASY estimates $\hat{\hat{\alpha}}_i$ and $\hat{\hat{\beta}}_{hi}$ are calculated as in the equations on top of page 18 of Jones and Novick (1972) but with omission of the factors $(1 - m^{-1})$.

d. The estimated intercepts variance $\hat{\tau}_\alpha$ is recalculated for the revised ASY estimates $\hat{\hat{\alpha}}_i$.

e. The ASY estimates $\hat{\hat{\phi}}_i$ for the residual variances are obtained from the LS estimates $\hat{\phi}_i$ in the following way:

$$p_i = 1/n_i - 1 \qquad (n_i = \text{sample size of } i^{th} \text{ group});$$

$$t_i = \log \hat{\phi}_i + p_i;$$

$$\bar{t} = m^{-1} \sum_{i=1}^{m} t_i;$$

$$\zeta = \sum_{i=1}^{m} (t_i - \bar{t})^2/(m - 1) - 2 \sum_{i=1}^{m} p_i/m;$$

$$\hat{\hat{\phi}}_i = \begin{cases} \exp\{(\zeta t_i + 2p_i\bar{t})/(\zeta + 2p_i)\} & \text{if } \zeta > 0; \\ \exp(\bar{t}) & \text{otherwise.} \end{cases}$$

One recognizes the weighted average character of such estimates, and it will be obvious that the averaging and weighting is carried out for the logarithm of the variance rather than for the variance itself.

7. The user is asked to provide a minimum value for the residual variance (PHIMIN), and for the prior parameter variance ($\tau_{\beta h}$), and he may use card 4 to choose wise zero values for the predictand and the predictors. Malfunctioning may occur, however, when such choices would inadvertently be made in total disagreement with the characteristics of the data.

On the basis of these conclusions, several program changes have been carried out. They are summarized below; where necessary a more detailed description and discussion is presented in the next sections.

A. Because of conclusions (1) and (2), the new MBREG program uses just one set of initial estimates. In most cases these will be the least squares (LS) solutions for the individual groups. The user may, however, introduce the ASY estimates obtained from MPREP, the final estimates obtained at an earlier run of MBREG, or any other set he desires to use. In the first two cases these estimates are produced in punched cards when desired, ready for introduction into MBREG. In the third case manipulation of the INIT parameter (see section 1) allows the use of intercept values corresponding to any scaling of the predictors.

B. The adaptation of the prior variances τ_α and $\tau_{\beta h}$ $(h = 1, 2, . . ., \ell)$ mentioned in conclusion (3) will be made in the MPREP program. This means ASY estimates will no longer be calculated in the MBREG program, which then requires less input and less preparatory calculations. More

For a convenient use of the new MBREG program, the new MPREP should produce punched output which can immediately serve as input to MBREG. From the input description in section 2, it can be seen that this means punching of the cards 1, 2, . . ., 5 (of which the user might want to adapt the parameter card to his wishes if he is not satisfied with the default values, and of which the ideal points scaling card is the only one containing new information). The input to MPREP would have reversed order of predictors and criterion on card 4, compared to BPREP, in order to be compatible with MBREG. The deck of SCP matrix cards can be taken from the MPREP input, including its format card for which 10A8 has been replaced by A8. The coefficient cards mentioned under 8 in section 1 should be punched by MPREP in the new format, omitting the estimated squared errors cards.

If ASY estimates are considered to be important, all steps (a) to (e) will have to be incorporated in the MPREP program. If not, there remains the problem that input card 9 of MBREG requires population variances estimates $\hat{\tau}_\alpha$ and $\hat{\tau}_{\beta h}$ $(h = 1, 2, \ldots, \ell)$. They could be printed and punched as in the present BPREP program without substantial changes if the revision of $\hat{\tau}_\alpha$ according to (d) is neglected. We shall discuss in section 6 the whole problem of the τ estimates; pending further investigation it will be left open whether it is worthwhile to incorporate a revised estimate of τ_α from the data into MPREP.

4. How Leaps Speed Up Convergence

This section tells how and why leaps have been included in the main program MBREG. An illustration of the argument is borrowed from the next section, in which the method will be applied to several data sets.

Table 1 (see appendix) describes in some detail the estimated values of the intercept α_1 (with predictors centered at the ideal point), the regression coefficients β_h ($h = 1, 2, 3, 4$) and the residual variance ϕ_1 for the first of 10 groups for which it was tried to predict grade point average from the four predictors English, Mathematics, Social Sciences, and Natural Sciences. The data set is described under the name "10HETERO" in section 5. During the iteration process, the parameter values change more or less systematically, after the first few steps; the improvements become smaller and smaller, and the parameters seem to undergo a monotone and decelerating movement toward the optimal values. A similar behavior is observed with any other group in the "10HETERO" data set, and with the groups of the other data sets that have been analyzed. This phenomenon is familiar to many numerical analysts, and has been described by Fischer (1974, p. 245) for the estimation of item difficulties in the logistic Rasch model.

Fischer also suggests a solution: the sequence of differences between values obtained at successive cycles can be approximated by a geometric progression. From three successive values one may estimate the ratio of two successive differences, and use this ratio to predict the

"asymptotic" value. Consider $10^5\beta_{11}$ in Table 1 at cycles 4, 5, and 6: the differences are 20 and 16, or more accurately, 20.03 and 15.50, with a ratio $\rho = 15.50/20.03 = 0.774$. The predicted asymptotic value is now $3206.94 + 15.50\ \rho/(1 - \rho) = 3259.78$. Although this is still far from the actual value of 3433 given in the last line of the table, the undisturbed iteration process needs eight more steps from cycle 6 until $10^5\beta_{11}$ reaches 3260. Similar calculations for the other parameters lead to predictions of 2007, 2083, 2082, 482, and 4803 for the other columns. A glance at Table 1 shows that this leads us astray for $10^4\alpha_1$ and $10^5\beta_{41}$, for which the direction of change is reversed at the eighth cycle; it slightly overestimates the change in $10^4\phi_1$ and underestimates the changes in the other parameters.

Not knowing the final values, one could continue a few more cycles from the values obtained by a "leap" after the sixth step. Such an alternation of cycles and leaps, with a few refinements to be described next, produces the estimation process described in Table 2. It is seen that the leap after cycle 14 already produces a higher criterion = log posterior density value than obtained in 50 iteration steps of the undisturbed process. This holds for both runs described in Table 2 (see appendix). A discussion of the difference between those runs is postponed until section 5.

Some qualifications and refinements are necessary for the satisfactory behavior of an alternation of leaps and iterations. We shall illustrate this by considering the parameter β_{32} in Table 3 (see appendix) (column marked β_{3i} for third predictor, row 2 in each block for second group), for cycles 4, 5 and 6. The successive differences are -6.02 and -8.95. The geometric ratio would be larger than unity, but it is certainly unsatisfactory to predict that the whole sequence of successive β_{32} values diverges to $-\infty$.

Another problem is met for α_7, where the successive differences are seen from Table 3 to be +1.82 and -.20. Does this imply a next difference of +.02 and then a still smaller negative one, as would follow in our geometric series model? Although this might be true, it is far more plausible that we have hit upon a point in the undisturbed iteration process where an initial increase is just changing into a stable decrease (cf. the column for $10^4\alpha_1$ in Table 1).

A third problem is the instability of the estimated ratio ρ, because of cross-influences from other parameter estimates and because of systematic deviations from the geometric model. When ρ is estimated from three cycles ending at cycle number 5, 6, 7, 8, 9, 10, respectively, for the parameter β_{11}, from a version of Table 1 with more decimals, one obtains .950, .774, .726, .814, .811, .848. Such behavior is certainly not atypical and it shows how rough an extrapolation from three cycles can be.

For a solution of these problems, three strategies were considered:

a. consider more than three cycles and use an extrapolation method geared to what was observed;

b. consider more than three cycles and make extrapolations only for those parameter values for which conformity to the geometric series model was observed;

c. use the ratio averaged across groups in the extrapolation.

There is empirical evidence that each of the strategies improves the prediction of the final parameter values, compared to the simple leaps described before. Although the data used in section 5 in our benchmark examples contained groups of rather different sample size and regression parameter values alongside with more homogeneous subgroups, there is a substantial similarity between groups when one considers the changes in parameter

values from cycle to cycle. The dominant behavior, to our surprise, was not a general regression to the mean (across groups) of the least squares solutions, but a shift in which some regression parameters (β_1, β_2, and β_4 in the "10HETERO" data of Tables 1-3) grew larger at the expense of others (β_3 in this data set). Although the risk of capitalizing on the peculiarities of some data sets should not be lightly dismissed, the evidence built up at this stage seems to indicate that a successful extrapolation of the iteration process is indeed possible.

All three strategies (a), (b), and (c), however, have a major drawback when considered in the light of the goal: inclusion of m-group regression into the CADA Monitor. They all involve a considerable amount of "bookkeeping" within the program, which might go as far as doubling the memory space and execution time required per iteration step. For CPU time, this is easily gained back because of the drastic reduction in the number of cycles required, but for use on medium sized computers the core limitations impose strong restrictions on the number of values that can be kept available during the iteration process. Moreover, there is room for doubt about the stability of an iteration cycle that might start with some parameter values very close to the optimal ones and others still very far away from it.

It was decided for this reason that "leaps" would be programmed for all parameters and groups simultaneously, and would be based on the individual geometric ratios calculated from the past three values. The factor $\rho/(1 - \rho)$ would be replaced by a fixed number SCH for cases where the past two differences had opposite signs, by a fixed number DCN for cases where the last difference was larger than its predecessor [or so little less that $\rho/(1 - \rho)$ would exceed DCN], and by zero if the last difference meant a relative change of less than 10^{-NDB} in the parameter value. The

numbers SCH, DCN, and NDB can be changed by the user of the program; their default values have been chosen as 20, 10, and 4, respectively, on the basis of experiments described in section 5.

That section also describes the empirical answer to the question of how often one should insert leaps between the normal iteration cycles. A first leap after 5 cycles followed by leaps after each 4 cycles seems to produce very fast convergence, but it is generally true that many other strategies lead to essentially the same criterion values in only a few extra steps.

As an occasional "bad leap" could produce a decrease of the criterion in some situations, the parameter values before the leap are stored, and the process restarts iteration from those values when the criterion value after the leap followed by one iteration is less than the criterion value before the leap. This "bad leap" provision should be distinguished from the "reduced step" provision (E) of section 2, which applies to decreases of the criterion occuring at other stages than just after leaps.

5. Experiences for Four Data Sets

At the time of the author's short visit to Iowa City, only one data set was directly available. It consists of the values of the observed Grade Point Average (predictand) and the ACT test scores English, Mathematics, Social Sciences, and Natural Sciences (four predictors) for the pupils of 22 junior colleges, out of which a 25 per cent sample of the 1968 data was obtained as described in Novick, Jackson, Thayer, and Cole (1971, 1972). This "25 per cent 1968" data set for all 22 colleges and all four predictors will be denoted as "22&4" in the sequel. It leads to

modal estimates in which the slopes for the fourth predictor are totally regressed.

Out of these 22 colleges, we selected the ones numbered 1, 2, 4, 9, 14, 16, 17, 20, 21, 22 because they led to widely differing slope values; this data set will be called "10HETERO" for obvious reasons, and its modal solution contains no parameters with total regression to the mean. The remaining 12 colleges form the data set "12HOMO," for which the second and fourth predictor show total regression. Finally, in the spirit of Jones and Novick (1972, p. 11), we analyzed all 22 schools with only the first and third predictor left, under the name "22&2."

For each of the four data sets, 50 iterations were carried out with the original BAYREG program. Although some common elements were found in the iteration behavior, see section 4, the data sets behaved rather differently in other respects. A parallel change for the slopes of the different groups was sometimes observed right from the start, sometimes only after some four to six iterations; sometimes such changes occurred for all slopes, sometimes only for a few. Convergence was disappointingly slow, especially for the full "22&4" data set and "10HETERO" but also for "22&2." There are enough differences in the behavior during iterations to use the four data sets as trial examples for improved convergence, pending further experiments on new data sets.

The parameters guiding the leap process were tried out in various combinations. The main conclusion is that the number of iterations, and therefore the computer time, required to reach a point at which the

criterion remains constant in five digits, is reduced to one third, sometimes even less, by the introduction of leaps. Many parameter choices including leaps after every fourth or fifth cycle are far better than the undisturbed process. The first leap seems to be most effective when taken after the fifth or sixth cycle.

Although the exact leaps + iterations behavior varies for different choices of the leap-governing parameters, many such choices lead to almost the same strong gain in speed. Even if the final criterion value at which the process stops, for one of the reasons given in section 2 under (F), is not quite the same, the modal estimates tend to vary only slightly from one run to the next run with different NDH, NDB, NCI, NCF, SCH, or DCN. A typical illustration is given in Table 4 (see appendix), where it is seen that the model estimates are almost the same for all lines except the last ones which pertain to the undisturbed iteration process. Sometimes, however, almost the same regression coefficients may still produce somewhat different outcome of a cross validation study, as will be discussed in section 6.

Since the main text of this report was written, the author obtained access to the BOVO MAVO 72/73 data from Jansen (1977) with twelve groups and two predictors, of which one has a negative ASY estimate for τ_β. The undisturbed iteration process has a log posterior density of -1056.9005 at cycle 4, -1056.5288 at cycle 5, -1056.4985 at cycle 6, and remains constant at -1056.4916 from cycle 15 onwards. A leap process with all default options jumps from cycle 5 to -1056.4915 at cycle 6. A leap from cycle 4 even jumps to -1056.4905 at cycle 5. Although convergence for these data was already fast, leaps again reduce compute time by a factor between 2 and 3.

6. The Road Ahead

This section contains a brief discussion of some points on which future work in the area might be concentrated.

1. As mentioned in conclusion (6) and shown in Table 4, many somewhat different parameter sets lead to very nearly the same value of the log posterior density. In most cases the goal will be good prediction in future applications. The experiences thus far seem to indicate that the use of two similar sets of regression coefficients, both close to the modal ones, produces somewhat unpredictable results at cross-validation: the quality of prediction changes sometimes very little, sometimes rather much. Obviously this also depends on the operationalization of "quality of prediction": mean squared error, absolute error, and correlation can be expected to have a more continuous behavior than zero-one or threshold loss functions. In the presence of outliers, however, it has been observed for some data sets used in cross-validation that even the former three criteria may react rather sharply to small changes in the regression equations. The average increase of prediction quality mentioned in earlier cross-validation studies is indeed an average, formed in most cases from a substantial improvement in some groups and a deterioration in a few others. Moreover, improvement in mean squared error sometimes goes with decreasing correlation or absolute error, or the other way around.
2. In the light of recent work by Smith (1977), it may be time to repeat the question of whether modal estimates are indeed the best end product of an m-group regression analysis. Although the answer may depend heavily on the particular circumstances and the careful specification of prior in-

formation, it may be worthwhile to explore in more detail whether there are other estimates which can be obtained without too much trouble, and whether modal estimates lead to better predictions than those others.

3. It remains to be investigated whether the models of Shigemasu (1976) can be incorporated in the general m-group regression program. Both the use of equal-slopes-unequal-intercepts and the relaxation of the exchangeability assumption could widen the scope of the method.

4. Most papers on m-group regression state that the matrix Σ occurring in the Lindley equations should be taken as a diagonal matrix with three times the a priori most probable values of the variances τ_β and $\tau_{\beta h}$ ($h = 1, 2, \ldots, \ell$) on the diagonal. As discussed in section 2, the computer programs BPREP and BAYREG take as "most probable values" the variances of the ASY (essentially model II) estimates, after revision for possible negative variance estimates. These estimates, however, are calculated from the data: their use means, strictly speaking, that the posterior distribution is no longer the product of prior and likelihood. The point is unimportant as long as the choice of the τ estimates has very little influence on the end result. Lindley (1969), after explaining that such estimates are necessary to prevent divergence, comments (page 8), "If these terms play an important role in the calculations described below, then this indicates that the sample is providing information comparable in magnitude with that available a priori: that is, very little."

Our problem is just this: in the five data sets studied, the choice of these terms turns out to have a strong influence on the iterative process leading to the modal estimates and on the final maximum of the log posterior density, and a weaker, but non-negligible, influence on the modal

estimates of the various regression parameters. It will be investigated in the next months whether this is also true for other data sets, and whether a choice $0 < \nu' < 1$ for the degrees of freedom of the Wishart distribution of the inverse of the regression parameters covariance matrix, diminishing the influence of the prior information, could solve the problem.

5. A further simplification of the program would be possible if a geometric ratio ρ derived from the past three values of β_*, the parameter averaged across groups, could be used, It may take more than four to six cycles before the ratios for different groups are sufficiently alike to make such averaging productive. It remains to be seen whether the ρ from β_*, common to all groups but different per predictor (or intercept or residual variance), should then be applied to the difference of β_* (this would do away with most of the arrays of size 25 by 5 now in the program) or to the individual differences per predictor and per group. A still further step in the same direction would be the replacement of the iteration process, after a number of cycles, by a step of the form $\alpha_{i,t+1} = \alpha_{i,t} + c_\alpha$; $\beta_{hi,t+1} = \beta_{hi,t} + c_h$ $(h = 1, 2, \ldots, \ell)$, and $\phi_{i,t+1} = \phi_{i,t} + c_\phi$, in which the constants c would be determined so as to maximize the log posterior density. Such a step, uniform across groups, could then be followed by a new set of iterations from its end-point. Research along these lines is now in progress. Its feasibility is illustrated by Table 5 (see appendix), which gives the differences between the values at cycle 6 and the model estimates for the two data sets. Especially for β_1, β_3, and β_4 with "10HETERO," and β_1 and β_2 with "22&2," where the most drastic changes occur, the differences do not differ too much across groups.

6. As stated in conclusion (7) of section 2, the program may produce some undesirable results when faced with unusual input data, e.g.,
   - when some predictor(s) or the predictand are scaled with an extremely large or extremely small scale unit,
   - when some predictor(s) or the predictand have a zero scale point very far from the actual values,
   - when all or some groups, possibly the small ones, produce an (almost) perfect fit to the least squares solution.

   Rescaling is advised in the first and second case; in the CADA Monitor version this burden could be taken away from the user by following a suggestion of Dr. Charles Lewis: let the program rescale all predictors to mean square deviation 1 around the ideal points, and rescale the predictand to zero mean and unit variance. In the third case, collection of more data or deletion of some predictors may reduce the danger of overfitting; a careful comparison of the least squares solution for the residual variances ϕ_i to the minimum PHIMIN is recommended.

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Appendix: Tables I-V

Table 1
Parameter values for the first group, and criterion-log posterior density, during the iteration process for the undisturbed iterations for the "10HETERO" data set described in section 5.

Cycle no.	Criterion	$10^4\alpha_1$	$10^5\beta_{11}$	$10^5\beta_{21}$	$10^5\beta_{31}$	$10^5\beta_{41}$	$10^4\phi_1$
1	79.3542	1993	3175	1687	2542	890	4948
2	85.0815	1949	3154	1916	2344	707	4757
3	85.5006	1968	3150	2001	2320	587	4767
4	85.5679	1987	3171	2044	2308	530	4779
5	85.5943	1998	3191	2064	2298	504	4788
6	85.6120	2003	3207	2074	2288	492	4793
7	85.6267	2005	3219	2079	2278	488	4797
8	85.6400	2005	3228	2081	2268	487	4799
9	85.6523	2004	3235	2082	2258	489	4800
10	85.6639	2003	3242	2083	2249	491	4801
15	85.7141	1994	3266	2085	2206	509	4802
20	85.7535	1985	3285	2086	2167	528	4802
25	85.7846	1977	3302	2087	2133	544	4802
30	85.8090	1969	3318	2088	2103	558	4802
35	85.8283	1963	3331	2088	2076	571	4802
40	85.8434	1957	3343	2089	2052	582	4802
45	85.8554	1952	3354	2090	2031	592	4802
50	85.8648	1948	3363	2090	2012	601	4802
55	85.8722	1944	3371	2091	1995	608	4802
60	85.8781	1941	3379	2091	1980	616	4802
65	85.8827	1938	3385	2091	1967	622	4802
70	85.8863	1935	3391	2092	1956	628	4802
75	85.8892	1932	3396	2092	1945	633	4802
80	85.8915	1930	3401	2092	1936	637	4802
85	85.8932	1928	3405	2093	1928	641	4802
90	85.8946	1927	3409	2093	1921	644	4802
95	85.8958	1925	3412	2093	1914	647	4802
*Max	85.8999	1915	3433	2094	1871	668	4802

*This line could have been obtained, in principle, by taking many more iteration cycles, but it was actually obtained at the 37th iteration of a leap process (Table 2).

Table 2

Parameter values for the first group, and criterion values for two runs of the speeded program applied to the "10HETERO" data set described in section 5 for NDB = 3, NDH = 5, NCI = 6, NCF = 4, DCN = SCH = 20, VGT = .5, NCY = 25 (first half) and NDB = 4, NDH = 6, NCI = 6, NCF = 4, DCN = 30, SCH = 20, VGT = .5, NCY = 50 (below the solid line).

Cycle No.	Criterion	$10^4\alpha_1$	$10^5\beta_{11}$	$10^5\beta_{21}$	$10^5\beta_{31}$	$10^5\beta_{41}$	$10^4\phi_1$
6	85.6120	2003	3207	2074	2288	492	4793
Leap	-	2007	3260	2083	2082	482	4803
7	85.7391	1981	3289	2095	2160	531	4798
10	85.7789	1978	3298	2087	2139	541	4801
Leap	-	1978	3375	2087	2006	602	4801
11	85.8298	1959	3345	2090	2057	582	4802
14	85.8480	1956	3347	2089	2044	586	4802
Leap	-	1956	3347	2089	1957	626	4802
15	85.8690	1940	3369	2092	1991	616	4802
18	85.8772	1940	3376	2091	1982	616	4802
Leap	-	1940	3376	2091	1925	630	4802
19	85.8862	1931	3392	2095	1948	634	4802
22	85.8900	1931	3397	2092	1942	635	4802
Leap	-	1931	3397	2092	1942	635	4802
23	85.8913	1930	3399	2093	1937	637	4802
25	85.8921	1929	3402	2093	1933	639	4802
6	85.6120	2003	3207	2074	2288	492	4793
Leap	-	2007	3260	2083	1980	482	4803
7	85.7515	1968	3309	2101	2114	555	4803
10	85.8135	1967	3319	2089	2096	562	4802
Leap	-	1944	3439	2088	1929	630	4802
11	85.8503	1953	3369	2090	2021	596	4802
14	85.8655	1948	3365	2090	2010	601	4802
Leap	-	1940	3377	2099	1904	666	4802
15	85.8804	1931	3390	2093	1952	636	4802
18	85.8889	1932	3395	2092	1946	633	4802
Leap	-	1932	3441	2092	1887	637	4802
19	85.8934	1927	3412	2094	1920	644	4802
22	85.8955	1926	3412	2093	1915	646	4802
Leap	-	1923	3412	2093	1878	652	4802
23	85.8971	1919	3420	2094	1895	659	4802
⋮							
37	85.8999	1915	3433	2094	1871	668	4802

Table 3
Fragment of the computer output describing the "leap" based on the values at cycles 4, 5 and 6 for the "10HETERO" data, see text. After a first line giving cycle number and criterion = log posterior density, there is a line for each group giving its parameter values at this cycle.

cycle	group	α_i	β_{1i}	β_{2i}	β_{3i}	β_{4i}	φ_i
1							
4		.85567919E+02					
4	1	.198722E+00	.317141E-01	.204410E-01	.230846E-01	.529719E-02	.477939E+00
4	2	.218253E+00	.233565E-01	.196312E-01	.225506E-01	-.192014E-01	.461179E+00
4	3	.300992E+00	.167197E-01	.324606E-01	.221822E-01	.648367E-03	.467857E+00
4	4	.189310E+00	.444036E-01	.328801E-01	.236572E-01	.263572E-01	.466738E+00
4	5	.241519E+00	.366613E-01	.338794E-01	.231760E-01	.845343E-02	.494629E+00
4	6	.158049E+00	.475703E-01	.346871E-01	.234329E-01	.106513E-01	.471864E+00
4	7	.148270E+00	.299882E-01	.249906E-01	.222501E-01	-.880845E-02	.460792E+00
4	8	.424079E+00	.226537E-01	.180609E-01	.222705E-01	.348202E-02	.466976E+00
4	9	.174106E-01	.463704E-01	.351141E-01	.236781E-01	.739955E-02	.481816E+00
4	10	.909752E-01	.364153E-01	.292766E-01	.232540E-01	.957513E-03	.471437E+00
5		.85594306E+02					
5	1	.199776E+00	.319144E-01	.206426E-01	.229780E-01	.503686E-02	.478776E+00
5	2	.219320E+00	.235883E-01	.197113E-01	.224904E-01	-.193533E-01	.461952E+00
5	3	.301490E+00	.170231E-01	.325541E-01	.221313E-01	.455729E-03	.468623E+00
5	4	.189980E+00	.446012E-01	.331198E-01	.234967E-01	.259523E-01	.467537E+00
5	5	.242117E+00	.368874E-01	.340232E-01	.230579E-01	.827359E-02	.495387E+00
5	6	.158808E+00	.477688E-01	.348564E-01	.232778E-01	.104335E-01	.472560E+00
5	7	.148452E+00	.302016E-01	.250832E-01	.221496E-01	-.893549E-02	.461403E+00
5	8	.424476E+00	.229538E-01	.182195E-01	.222256E-01	.322406E-02	.457695E+00
5	9	.181830E-01	.465688E-01	.352922E-01	.235120E-01	.721302E-02	.482508E+00
5	10	.915989E-01	.366290E-01	.294173E-01	.231254E-01	.786180E-03	.472107E+00
6		.85612001E+02					
6	1	.200276E+00	.320694E-01	.207384E-01	.228754E-01	.491976E-02	.479315E+00
6	2	.219788E+00	.237567E-01	.197500E-01	.224009E-01	-.194162E-01	.462452E+00
6	3	.301645E+00	.172209E-01	.325960E-01	.220454E-01	.365874E-03	.469111E+00
6	4	.190333E+00	.447654E-01	.332251E-01	.233780E-01	.257541E-01	.468048E+00
6	5	.242367E+00	.370557E-01	.340872E-01	.229523E-01	.819379E-02	.495879E+00
6	6	.159203E+00	.479215E-01	.349309E-01	.231612E-01	.103308E-01	.473006E+00
6	7	.148432E+00	.303495E-01	.251281E-01	.220507E-01	-.898840E-02	.461789E+00
6	8	.424614E+00	.231623E-01	.182934E-01	.221417E-01	.310968E-02	.468152E+00
6	9	.185756E-01	.467186E-01	.353748E-01	.233923E-01	.712385E-02	.482953E+00
6	10	.918755E-01	.367861E-01	.294839E-01	.230167E-01	.704763E-03	.472538E+00
AFTER LEAP FROM CYCLE	6						
6	1	.200728E+00	.325978E-01	.208252E-01	.208222E-01	.482403E-02	.480293E+00
6	2	.220153E+00	.242042E-01	.197859E-01	.206120E-01	-.194606E-01	.463370E+00
6	3	.301645E+00	.175909E-01	.326299E-01	.203269E-01	.287323E-03	.469975E+00
6	4	.190722E+00	.455724E-01	.333077E-01	.230417E-01	.255639E-01	.468949E+00
6	5	.242547E+00	.379460E-01	.341383E-01	.220482E-01	.813012E-02	.495879E+00
6	6	.159632E+00	.484315E-01	.349893E-01	.228098E-01	.102390E-01	.473006E+00
6	7	.148432E+00	.306840E-01	.251704E-01	.200718E-01	-.902616E-02	.461789E+00
6	8	.424614E+00	.236360E-01	.183580E-01	.204640E-01	.301857E-02	.468152E+00
6	9	.189817E-01	.471812E-01	.354462E-01	.230829E-01	.704218E-02	.482953E+00
6	10	.920960E-01	.372226E-01	.295440E-01	.224268E-01	.631044E-03	.472538E+00

Table 4

Results of several runs for the "10HETERO" data, first group. The last column gives the cycle number at which the result was obtained, followed by "E" if iteration ended there and by "I" if it is an intermediate result.

Criterion	$10^4\alpha_1$	$10^5\beta_{11}$	$10^5\beta_{21}$	$10^5\beta_{31}$	$10^5\beta_{41}$	$10^4\phi_1$	NDB	NDH	NCI	NCT	DCN	SCH	CYCLE
85.8999	1915	3433	2094	1871	668	4802	4	6	6	4	30	20	37E
85.8989	1919	3425	2094	1889	659	4802	4	5	4	4	20	20	31E
85.8984	1920	3421	2094	1894	658	4802	4	6	6	4	30	20	25E
85.8973	1923	3418	2093	1904	652	4802	4	6	5	4	30	20	25E
85.8968	1923	3415	2093	1907	651	4802	3	5	6	6	15	20	35E
85.8967	1922	3418	2093	1904	653	4802	4	5	4	4	20	20	25I
85.8949	1925	3405	2096	1919	646	4802	3	5	6	6	15	20	25I
85.8938	1928	3406	2093	1925	642	4802	4	5	6	4	15	20	29E
85.8924	1929	3402	2093	1932	640	4802	4	5	6	4	15	20	25I
85.8921	1929	3402	2093	1933	639	4802	3	5	6	4	20	20	25E
85.8916	1930	3401	2092	1936	637	4802	4	5	6	4	11	20	25E
85.8648	1948	3363	2090	2012	601	4802	undisturbed						50E
85.7846	1917	3302	2087	2133	544	4802	undisturbed						25I
85.5679	1987	3171	2044	2308	529	4779	undisturbed						4I
79.3542	1993	3175	1687	2542	890	4948	undisturbed						1I

Table 5

Increments, per group of the "10HETERO" and "22&2" data sets, between parameter values at cycle 6 and modal estimates (cf. Table 2). Note similarity of increments across groups [see (5) of section 6].

"10HETERO" Data Set

	$10^4\alpha_i$			$10^5\beta_{1i}$			$10^5\beta_{2i}$		
i	CY=6	modal	incr.	CY=6	modal	incr.	CY=6	modal	incr.
1	2003	1915	-88	3207	3433	226	2074	2094	20
2	2198	2110	-88	2376	2633	257	1975	1965	-10
3	3016	2906	-110	1722	1962	240	3260	3262	2
4	1903	1847	-56	4477	4716	239	3322	3369	47
5	2424	2324	-100	3706	3970	264	3409	3443	34
6	1592	1524	-68	4792	5031	239	3493	3518	25
7	1484	1349	-135	3035	3257	222	2513	2545	32
8	4246	4174	-72	2316	2561	245	1829	1813	-16
9	1858	1283	-575	4672	4899	227	3537	3576	29
10	916	829	-87	3679	3905	226	2948	2998	50

(continued)

	$10^5\beta_{3i}$			$10^5\beta_{4i}$			$10^4\phi_i$		
i	CY=6	modal	incr.	CY=6	modal	incr.	CY=6	modal	incr.
1	2288	1871	-417	492	668	176	4793	4802	9
2	2240	1827	-413	-1942	-1788	154	4625	4633	8
3	2205	1793	-412	37	197	160	4691	4691	4
4	2338	1915	-423	2575	2726	151	4680	4687	7
5	2295	1879	-416	819	984	165	4959	4967	8
6	2316	1897	-419	1033	1158	125	4730	4734	4
7	2205	1797	-408	-899	-748	151	4618	4616	-2
8	2214	1801	-413	311	482	171	4682	4685	3
9	2339	1921	-416	712	835	123	4830	4836	6
10	2302	1887	-415	70	191	121	4725	4731	6

Table 5 (con't.)

"22&2" Data Set

i	$10^4\alpha_i$ CY=6	modal	incr.	$10^5\beta_{1i}$ CY=6	modal	incr.	$10^5\beta_{2i}$ CY=6	modal	incr.	$10^4\phi_i$ CY=6	modal	incr.
1	624	554	-70	3996	4309	313	3268	2910	-358	4753	4773	20
2	1128	1076	-52	3534	3874	340	3136	2777	-359	4614	4633	19
3	2181	2115	-66	4643	4986	343	3244	2884	-360	4720	4737	17
4	2385	2296	-89	3298	3629	331	3047	2692	-355	4727	4745	18
5	1287	1210	-77	4333	4668	335	3210	2859	-351	4636	4651	15
6	-1852	-1895	-43	3646	3942	298	3246	2892	-354	4798	4818	20
7	1114	1063	-51	4454	4784	330	3283	2918	-365	4655	4673	18
8	1728	1659	-69	3944	4285	341	3251	2887	-364	4661	4682	21
9	635	597	-38	4779	5111	332	3386	3022	-364	4678	4698	20
10	4253	4111	-142	4398	4758	360	3202	2835	-367	4686	4705	19
11	5821	5740	-81	4508	4888	380	3159	2788	-371	4695	4711	16
12	5712	5590	-122	4367	4760	384	3126	2759	-367	4587	4603	16
13	983	888	-95	3713	4043	330	3155	2798	-357	4676	4692	16
14	1318	1242	-76	4603	4940	337	3342	2983	-359	4976	4995	19
15	1227	1163	-64	4175	4516	341	3244	2886	-358	4634	4651	17
16	739	713	-26	5103	5430	327	3338	2977	-361	4755	4770	15
17	142	20	-122	3728	4026	298	2965	2631	-334	4628	4636	8
18	3387	3258	-129	4172	4531	359	3175	2817	-358	4664	4681	17
19	5375	5268	-107	4622	5005	383	3224	2956	-368	4666	4684	18
20	3994	3924	-70	4224	4594	370	3115	2758	-357	4647	4661	14
21	-1763	-1795	-32	4543	4844	301	3379	3028	-351	4854	4871	17
22	-606	-668	-62	4161	4470	309	3299	2948	-351	4715	4731	16

University Memo

To: All of University of Iowa Students and Employees

From: University Business Office

Any charges made to your University Account will be subject to the following terms:

Agreement and Disclosure Statement

Participation in the University's Open End Credit Plan is on the following terms and conditions:

1. The University, from time to time and in its sole discretion, may permit purchase of goods or services and defer payment thereof.

2. Payment shall be made as follows: On or before fifteen days after the first of each month, a minimum periodic payment at the University (by mail or otherwise) of the New Balance shown on the periodic statement for that billing is required.

3. The University may, at any time and in its sole discretion, terminate this Plan or revoke any or all privileges under the Plan. In any event, the Plan will terminate automatically upon ceasing either to be registered as a student or employed by the University.

4. Obligations shall be in default if:

a) any payment required by this agreement is past due more than ten days, or

b) Any representation made to induce extension of credit under this Plan proves untrue, any covenant of this agreement is breached, or the student or employee becomes insolvent or bankrupt so as to materially impair the prospect of paying obligations under this Plan.

In the event of a default (and after the University has given any notices and observed any waiting period which may be required by law) the entire balance under this Plan shall become due and payable at the option of the University.

5. Failure to pay the monthly periodic payment when due may subject a student to cancellation of registration. There is a $10.00 charge for the first reinstatement of registration each semester or session. Reinstatement is usually refused a second time.

In Case of Errors or Inquiries About Your Bill

The Federal Truth in Lending Act requires prompt correction of billing mistakes.

1. If you want to preserve your rights under the Act, here's what to do if you think your bill is wrong or if you need more information about an item on your bill:

a. Do not write on the bill. On a separate sheet of paper write [*Alternate:* Write on the bill or other sheet of paper] (you may telephone your inquiry but *doing so will not preserve your rights under this law*) the following:

i. Your name and account number (if any).

ii. A description of the error and an explanation (to the extent you can explain) why you believe it is an error.

If you only need more information, explain the item you are not sure about and, if you wish, ask for evidence of the charge such as a copy of the charge slip. Do not send in your copy of a sales slip or other document unless you have a duplicate copy for your records.

iii. The dollar amount of the suspected error.

iv. Any other information (such as your address) which you think will help the creditor to identify you or the reason for your complaint or inquiry.

b. Send your billing error notice to the address on your bill which is listed after the words: "Send Inquiries To:."

Mail it as soon as you can, but in any case, early enough to reach the creditor within 60 days after the bill was mailed to you. If you have authorized your bank to automatically pay from your checking or savings account any credit card bills from that bank, you can stop or reverse payment on any amount you think is wrong by mailing your notice so the creditor receives it within 16 days after the bill was sent to you. However, you do not have to meet this 16-day deadline to get the creditor to investigate your billing error claim.

2. The creditor must acknowledge all letters pointing out possible errors within 30 days of receipt, unless the creditor is able to correct your bill during that 30 days. Within 90 days after receiving your letter, the creditor must either correct the error or explain why the creditor believes the bill was correct. Once the creditor has explained the bill, the creditor has no further obligation to you even though you still believe that there is an error, except as provided in paragraph 5 below.

3. After the creditor has been notified, neither the creditor nor an attorney nor a collection agency may send you collection letters or take other collection action with respect to the amount in dispute; but periodic statements may be sent to you, and the disputed amount can be applied against your credit limit. You cannot be threatened with damage to your credit rating or sued for the amount in question, nor can the disputed amount be reported to a credit bureau or to other creditors as delinquent until the creditor has answered your inquiry. *However, you remain obligated to pay the parts of your bill not in dispute.*

4. If it is determined that the creditor has made a mistake on your bill, you will not have to pay any finance charges on any disputed amount. If it turns out that the creditor has not made an error, you may have to pay finance charges on the amount in dispute, and you will have to make up any missed minimum or required payments on the disputed amount. Unless you have agreed that your bill was correct, the creditor must send you a written notification of what you owe; and if it is determined that the creditor did make a mistake in billing the disputed amount, you must be given the time to pay which you normally are given to pay undisputed amounts before any more finance charges or late payment charges on the disputed amount can be charged to you.

5. If the creditor's explanation does not satisfy you and you notify the creditor *in writing* within *10* days after you receive his explanation that you still refuse to pay the disputed amount, the creditor may report you to credit bureaus and other creditors and may pursue regular collection procedures. But the creditor must also report that you think you do not owe the money, and the creditor must let you know to whom such reports were made. Once the matter has been settled between you and the creditor, the creditor must notify those to whom the creditor reported you as delinquent of the subsequent resolution.

6. If the creditor does not follow these rules, the creditor is not allowed to collect the first $50 of the disputed amount and finance charges, even if the bill turns out to be correct.

7. If you have a problem with property or services purchased with a credit card, you may have the right not to pay the remaining amount due on them, if you first try in good faith to return them or give the merchant a chance to correct the problem. There are two limitations on this right.

a. You must have bought them in your home state or if not within your home state within 100 miles of your current mailing address; and

b. The purchase price must have been more than $50.

However, these limitations do not apply if the merchant is owned or operated by the creditor, or if the creditor mailed you the advertisement for the property or services.

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